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Hydraulic transients for a pipe line network of treated effluent rising main using SAP 2R

Ajmal Hussain[a], Muhammad Mustafa[b], Hazi Mohammad Azamathulla[c,*] and M. R. Sadique[a]

[a] Department of Civil Engineering, Zakir Hussain College of Engineering & Technology, AMU, Aligarh 202002, India
[b] Infrastructure Planner, Hydraulic Modeller Water and Wastewater Designer, CH2M Saudi Limited (A Jacobs Group Company), Jeddah, Kingdom of Saudi Arabia
[c] Department of Civil Engineering, The University of the West Indies at Saint Augustine, Port of Spain, Trinidad
*Corresponding author. E-mail: hazi.azamathulla@sta.uwi.edu

HMA, 0000-0002-5436-4147; MRS, 0000-0002-9570-6801

ABSTRACT

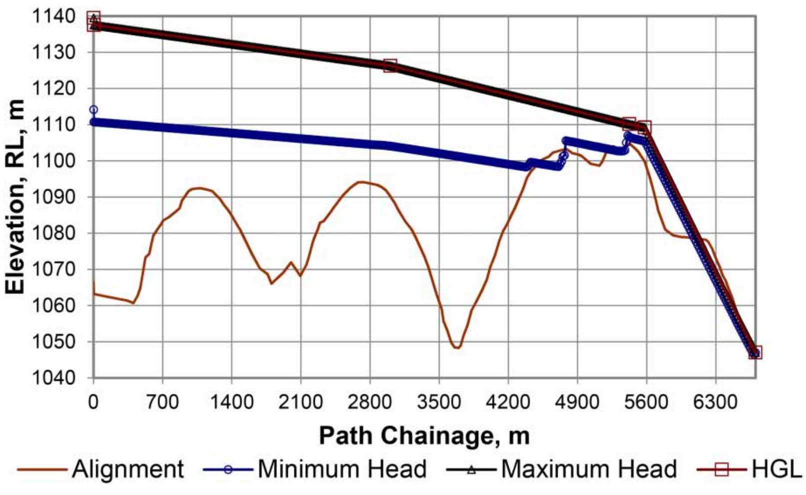
Hydraulic transients occur as a direct result of rapid variations of flow field in pressurized systems. The change in velocity from valve closures or pump operations causes pressure surges that are propagated away from the source throughout the pipeline. The associated pressure changes during a transient period are quite large and occur quickly (within a few seconds). It should also be noted that when the maximum pressures exceed the bar ratings (mechanical strength) of the piping material, failure can occur. Similarly, if the minimum pressure drops below the vapour pressure of the fluid, cavitation can occur. The purpose of the present study is to model and simulate the hydraulic transients in a pipeline network system of the treated effluent rising main of Mpophomeni sanitation scheme using SAP 2R. A total of five scenarios were simulated using different combinations. The simulation results show that the transient pressures in the pipeline exceeded the bar rating of the pipe where the bursts or cavitation may occur for the simulated scenario, but transient pressures were reduced to a safe limit after providing water hammer protection devices.

Key words: cavitations, hydraulic transient, pipe network, SAP2R, water hammer

HIGHLIGHTS

- Water hammer analysis is done on a pipe line network system using SAP2R.
- Water hammer pressure is observed and discussed on various sections of the pipeline.
- The method of characteristics is used for solving the hydraulic transient equations.
- Water hammer protection devices are suggested to keep transient pressure to a safe limit.

GRAPHICAL ABSTRACT



1. INTRODUCTION

As the infrastructure for cities and industries is fast growing, the water supply projects are increasing. As the capital costs are rising steeply, optimal engineering designs with reliability are the need of the hour. The design of surge protection systems and their analysis play an important role in this direction. Hydraulic transients, also known as pressure surges or water hammers, are time-varying phenomena that occur when the equilibrium of steady flow in a system is disturbed by a change of flow that occurs over a relatively short time period (Ghidaoui & Kolyshkin 2001; Nikpour *et al.* 2014; Nolan 2017). Water-hammer or surge is a phenomenon occurring in closed conduit or pipe flows, associated with rapid changes in discharge in the pipe. The water-hammer wave is propagated at acoustic speed, which varies with the material and wall thickness of the pipe. When a single pump fails with multiple pumps in parallel operation, the speed of the failing pump rapidly drops, along with discharge from this pump. The other running pumps get overloaded to partly compensate for the reduced discharge.

The pressure at the delivery manifold reduces, but this reduction is not very significant in view of the head generated by the running pumps. The greater the number of running pumps, the less pressure reduction in the manifold. Because of this, reverse flow occurs very rapidly through the failing pump. Typically, flow reversal may take place in 1–3 sec.

The transient state of the flow from time of closure until a new steady-state condition is established is complex due to pressure surges that propagate away from the valve (Wylie & Streeter 1993). Energy losses mainly due to friction cause the transient pressure waves to decay until a new steady state is established. In general, transient events are usually most severe at control valves, pump stations, high-elevation areas, and remote locations far from overhead storage tanks (Raghuveer 2014).

Thus, surge control is extremely important for designing, operation, and protection of hydraulic systems (Ghidaoui & Kolyshkin 2001). However, in practice, sometimes it is impossible to analyze them all due to time and budget constraints. Therefore, empirical guidelines can be used to determine whether a complete transient analysis is required (Jones *et al.* 2006).

Bergant *et al.* (2006) has presented a comprehensive survey of laboratory tests and field measurements for water hammer with column separation. Deshmukh (2014) presented a hydraulic transient analysis with Bentley HAMMER v8i and validated the results of the Kolar water pipeline with manual calculations. The results obtained matched fairly well with the manual results for the same case. Ghidaoui *et al.* (2005) provide both a historical perspective and review of water hammer theory and an overview of recent developments in this field of fluid mechanics. Emadi & Solemani (2011) investigated the effect of parameters such as pipe diameter, thickness, moment of inertia, and temperature on maximum water hammer in the context of Kuhrang Pumping Station. Water hammer is one of the destructive hydraulic phenomena, occurring in the form of a rapid pressure wave, which is propagated in the pipeline and leads to severe damage (Nikpour *et al.* 2014). Among the approaches proposed to solve the single-phase (pure liquid) water hammer equations are the Method of Characteristics (MOC), Finite Differences, Wave Characteristic Method, Finite Elements, and Finite Volume (Asli *et al.* 2010). One difficulty that commonly arises relates to the selection of an appropriate level of time step to use for the analysis. In general, the smaller the time step, the longer the run time but the greater the numerical accuracy.

A variety of commercial software is available for simulating hydraulic transients and can be used to design sophisticated pipeline networks and research studies. The present Surge Analysis Program (SAP) is an offshoot of the software developed in-house at the Indian Institute of Science and used to design surge protection system for a number of projects. This basic software has evolved over a period of 20 years and gradually incorporated several design options and system complexities. As a result, this basic software has developed to a stage where practically any complex situation with regard to surges in pumping or gravity mains can be analysed. However, this basic software works completely with processed data using only non-dimensional variables. SAP provides results through several graphical output plots. Among these graphical plots, the basic one is the plot of minimum and maximum piezometric heads and the pipeline's longitudinal alignment and HGL.

Its calculation engine is based on the Method of Characteristics (MOC). This is perhaps among the most popular methods used to solve hydraulic transient equations. The MOC (especially for a constant wave speed) is superior compared to other methods, especially for capturing the location of steep wavefronts, illustration of wave propagation, ease of programming, and efficiency of computations. SAP2R uses the Method of Characteristic to solve non-linear differential equations, which have the following form (Evangelisti 1969; Fox 1977; Streeter 1967, 1972):

$$\frac{dV}{dt} + \frac{1}{\rho c} \cdot \frac{dP}{dt} + \frac{f|V|V}{2D} = 0 \tag{1}$$

$$a^2 \frac{\partial V}{\partial s} + \frac{1}{\rho} \frac{\partial p}{\partial t} = 0 \tag{2}$$

Solution of the above equations using MOC will be

$$C+: \frac{a}{gA}(Q_p - Q_{i-1}) + (H_p - H_{i-1}) + \frac{f\Delta x}{2gDA^2} Q_p |Q_{i-1}| = 0 \tag{3}$$

$$C-: \frac{a}{gA}(Q_p - Q_{i+1}) - (H_p - H_{i+1}) + \frac{f\Delta x}{2gDA^2} Q_p |Q_{i+1}| = 0 \tag{4}$$

where, Q is the fluid flow, H is the hydraulic grade line elevation, D is the inner diameter, t is the time, V is the velocity, ρ is the density of water, g is acceleration due to gravity, A is the cross-sectional area, and subscript i and p correspond to node.

For ease of solution, the velocity terms of Equations (1) and (2) are converted into discharge terms. Equations (1) and (2) are quasi hyperbolic equations and the general method of solving MOC is applied in Equations (3) and (4). However, in this case, the upstream reservoir boundary conditions and pump conditions have been used for MOC equations (Bergant *et al.* 2012). The compatibility of c+ line Q_{i-1} has a known value at the upstream section, and H_{i-1} also has a known value at the upstream section. Similarly, for c- line Q_{i+1} is known and for H_p, due to pump shut down, considering the initial value is equal to zero.

There are many boundary conditions considered for the solution, requiring reservoirs, pumps, pipeline branches, dead ends, and so on. This method saves time and reduces the possibility of mistakes, saves time and reduces the possibility of mistakes that may occur while copying data to the software. The flowchart for systematic surge protection offered in Figure 1 summarizes a comprehensive procedure for identifying the worse loading case and for suggesting the appropriate transient protection.

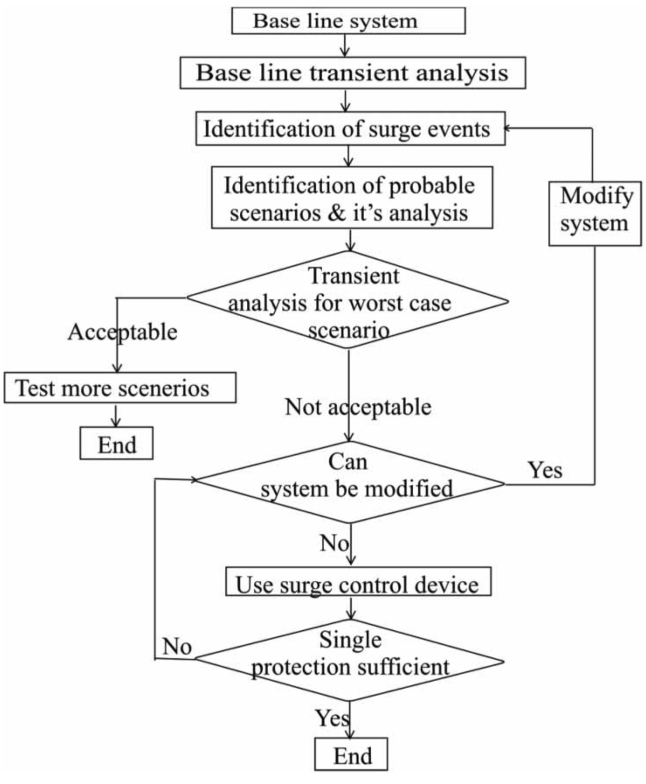


Figure 1 | Flow chart for systematic surge protection.

2. SYSTEM DESCRIPTION

The present study involves assessment of possible transient events which can occur in the proposed treated effluent rising main of Mpophomeni sanitation scheme, South Africa, traversing a path of approximately 6,750 m through a terrain consisting of various peaks and valleys as depicted in longitudinal ground profile (Figure 2(a)). Schematic representation of the transmission main system is shown in Figure 2(b). The network of treated effluent rising main consists of following major units (Mpophomeni Wastewater Works 2013; Water & Sanitation Africa 2013).

Pumping Station: The pumping station consists of three pumps with two working, and each with an individual capacity of supplying a flow rate of 250 m^3/h up to a head of 76 m. Each pump has a rated power of 90 kW at 1,450 rpm.

Rising Main Network: The rising main consists of PVC class 12 pipe of diameter 400 mm for the first 3,000 m (up to Node id N_2) and thereafter the pipe network is separated into two parallel pipes of 250 mm diameter and 315 mm up to node id N_3, then a small pipe length of 400 mm diameter between N_3 and N_4, then the pipe diameter reduces to 315 mm and continues till the tail end (N_5) discharging into the receiving chamber (Figure 2(b)).

Air Valves: The existing system is proposed with seven double-acting air valves of diameter 80 mm at mentioned locations shown in Table 1.

The pump station is located at zero chainage and the rising main culminates in free fall at a receiving chamber at the tail end. As evident from Figure 2(a), three major peaks and low points are observed along the network path. The network will be acting under gravity after a chainage of around 5,500 m. The undulating topography of the rising main path exposes the network to the risk of surge occurrences in events of power failure, the sudden closure of valves etc.

Generally speaking, a surge analysis is recommended if a system has one of the following cases (Jones, G.M.):

- Pumping system with a total dynamic head (TDH) larger than 14 meters (m) or 50 ft, and a flow greater than 115 cubic metres per hour (m^3/h) or 500 gallons per minute (gpm).
- Any pressurized pipe with a diameter greater than 200 mm (8 in.) and a length longer than 300 m (1,000 ft).
- Any system where column separations can occur, such as systems with knees (points where the gradient reduces) or high points, or pressurized pipelines with a steep gradient greater than 100 m (300 ft) followed by a long, shallow gradient.

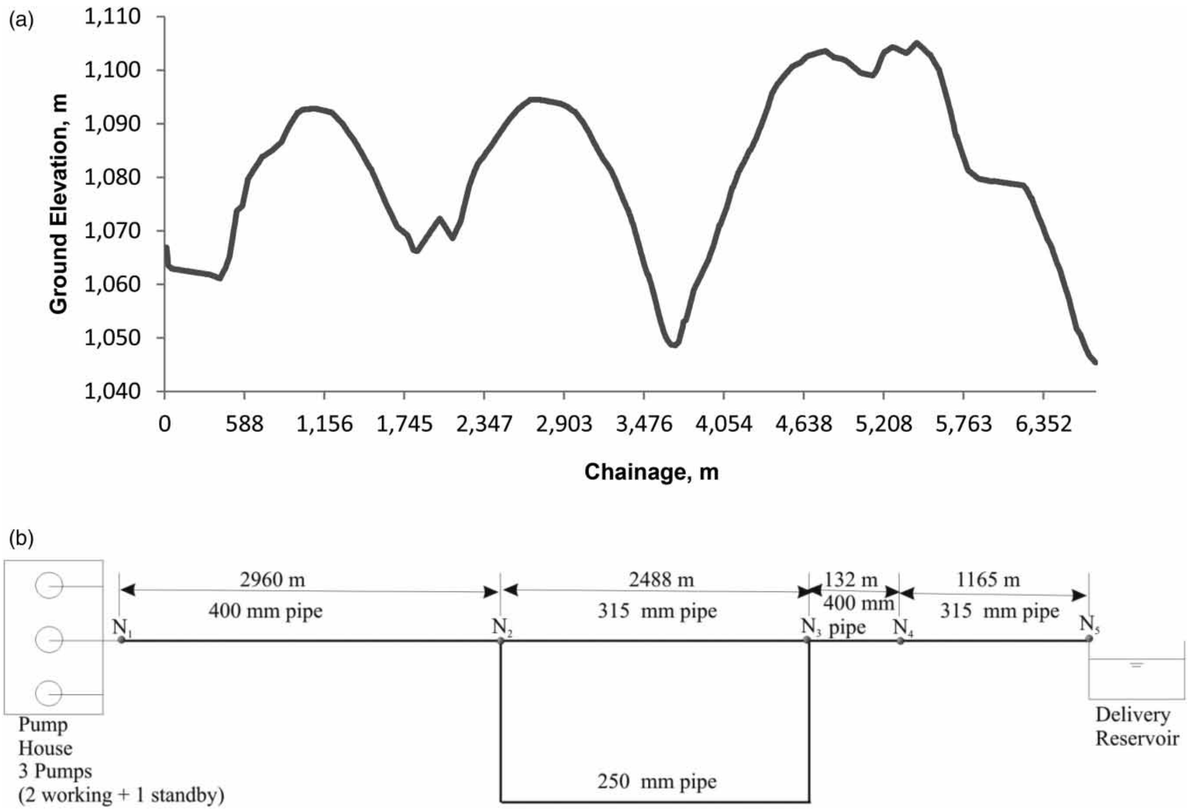


Figure 2 | (a) Longitudinal ground profile of rising main path. (b) Schematic representation of the main transmission system.

Table 1 | Originally proposed chainages of air valves

Valve No.	Chainage, m
AV1	1,080
AV2	1,998
AV3	2,729
AV4	4,786
AV5	5,255
AV6	5,580.25
AV7	5,957.56

Thus, a surge analysis is required to evaluate and assess the occurrence of surge pressures (upsurge and downsurge) within the network and remedial measures should be recommended to contain the surge effects.

Although analysis is carried out considering the entire network, as described in the above section, the main network under consideration is the main line with a diameter of 315 mm between node id N_2 and N_3 as the branch line of 250 mm can be isolated from the network. Thus all the results are presented as when the branch line of 250 mm is active and when the branch line of 250 mm is isolated from the network. The detailed descriptions of model settings for surge analysis are provided in Table 2.

3. RESULTS AND DISCUSSIONS

This section of the present study compiles the outcomes of analysis conducted under various scenarios as identified in the preceding sections.

3.1. Scenario 1: baseline network under steady state conditions

Firstly, a baseline run of the network composed of just the pumps and the pipeline without any air valves is conducted to identify the baseline scenario; that is, the network running under a steady state without any transient event. The hydraulic modelling of the system is completed to confirm the design of pumping and piping systems, i.e., checking whether the system is delivering the required heads at the required delivery points.

In the present case, there are two working pumps that supply treated effluent. During the steady state condition, the pump runs at the rated speed of 1,450 rpm. The baseline scenario is computed for two configurations: first, when the entire network is operating and second, when the 250 mm line between node N_2 and N_3 is isolated from the network. The graphical representation of the same is presented in Figures 3(a), 3(b), 4(a) and 4(b) under steady-state condition. Figures 3(a) and 4(a) shows that HGL is above the whole network; that is, the systems are delivering the required heads at the required delivery points when the entire network is functional and when the 250 mm line is isolated, respectively. Figures 3(b) and 4(b) show the pressure under steady-state condition for when the entire network functional and when 250 mm line is isolated, respectively.

Table 2 | Surge analysis model settings

Pressure wave speed	The wave speed varies from 340 m/s to 500 m/s for uPVC pipes of diameter 400 mm (18.4 mm wall thickness) and 315 mm (14.5 mm wall thickness)
Critical time period	For the analysis it is assumed that the check valves installed at the pump closes after 5 s of power failure, which is below the critical time period.
Liquid properties	Because the pumped fluid in the system is drinking water, a temperature of 20 °C and a specific gravity of 1.0 are assumed.
Vapor pressure	An approximate vapor pressure of –10.0 m (– 14.2 psi, –32.8 ft) is used for water systems at typical temperatures and pressures. Most of the pipelines are designed to sustain vapor pressure up to −10 m.
Elevations	Extremely important in hydraulic transient modeling. Therefore, defining a pipeline profile is a key requirement before undertaking any hydraulic transient analysis. The pipeline profile used for the analysis is the pipe top level.

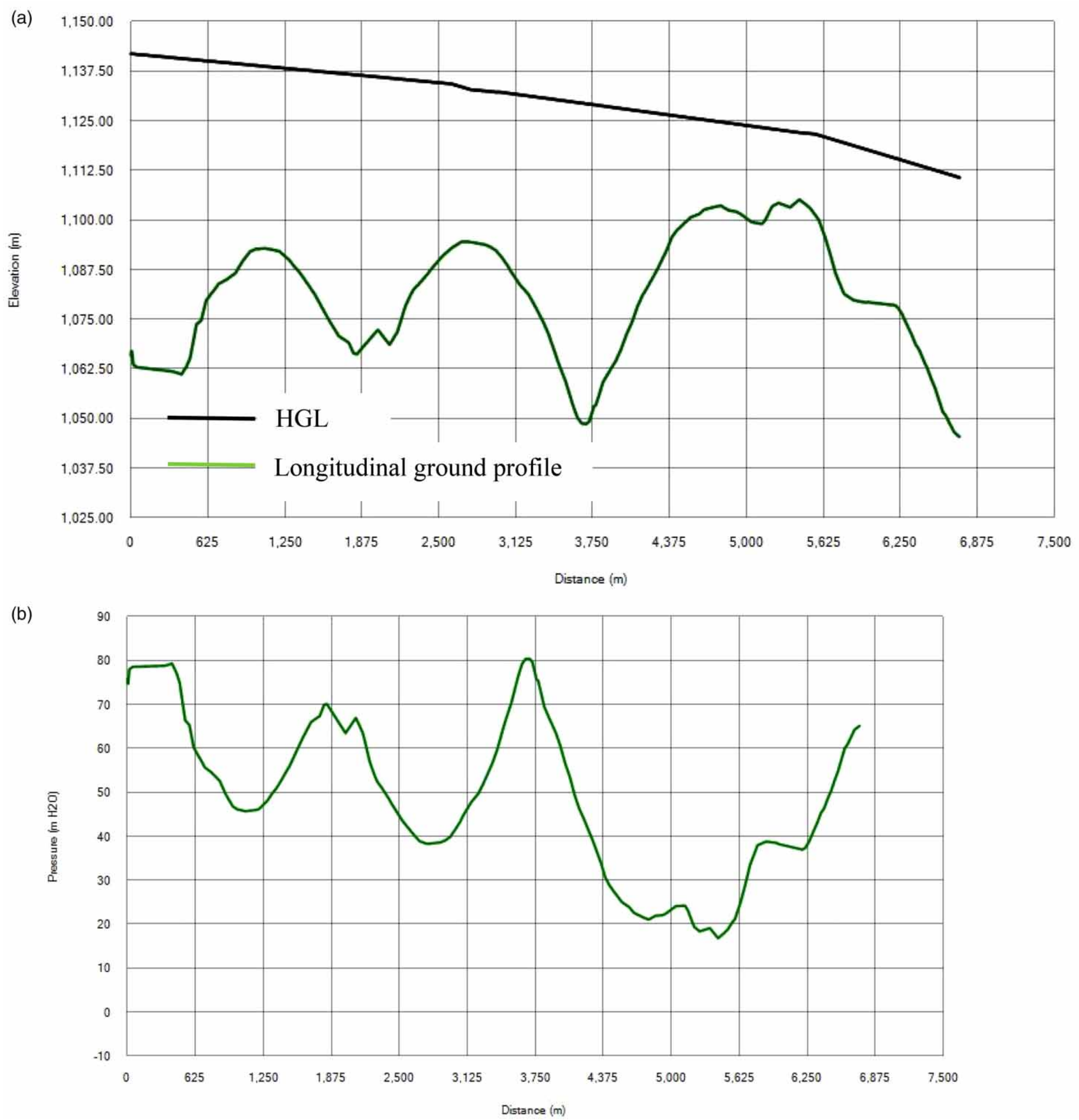


Figure 3 | (a) HGL under steady state condition in the system–entire network functional. (b) Pressure under steady-state condition in the system–entire network functional.

From Figures 4(a) and 4(b), the pressure profile is almost similar for both scenarios with a maximum pressure of 79 m H_2O and a minimum pressure of 5 m H_2O occurring in the system during the steady state operation. These HGL and pressure graphs serve as the reference line for other analysis carried henceforward.

3.2. Scenario 2: surge analysis on baseline network

In the second stage, a surge analysis is carried out on the baseline network for Scenario 2 (i.e., without any air valves) and identified the critical locations from the surge point of view. The most critical transient event in the system will be the power failure of both pumps. So, hydraulic transient analysis has been carried out for the system without provision of any air valve and protection device, under the power failure of both the pumps. Therefore, Scenario 2 is further subdivided into two sub scenarios as below.

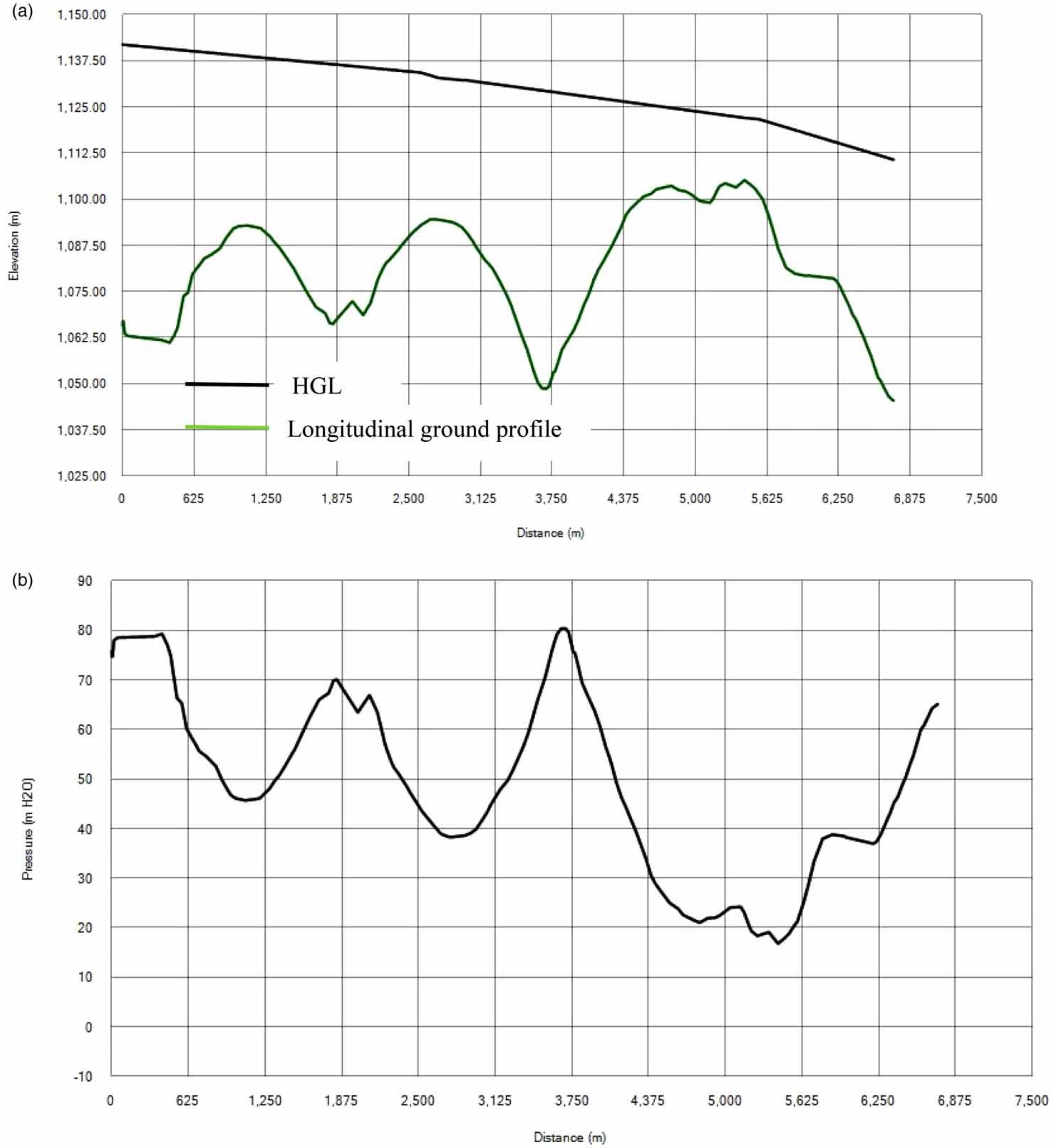


Figure 4 | (a) HGL under steady state condition in the system – 250 mm line isolated. (b) Pressure under steady state condition in the system-250 mm line isolated.

3.2.1. Sub scenario 2a: when the entire network is active

As seen from Figure 5, when the entire network is functional a slight upsurge is evident around the chainage 5,260 m, but its magnitude is very much less and is well within the operational limits of the proposed pipe class. However, considerable downsurge is evident over the entire stretch of the pipe, the magnitude of downsurge is well below water vapour pressure and thus commands remedial measures to be provided on the network.

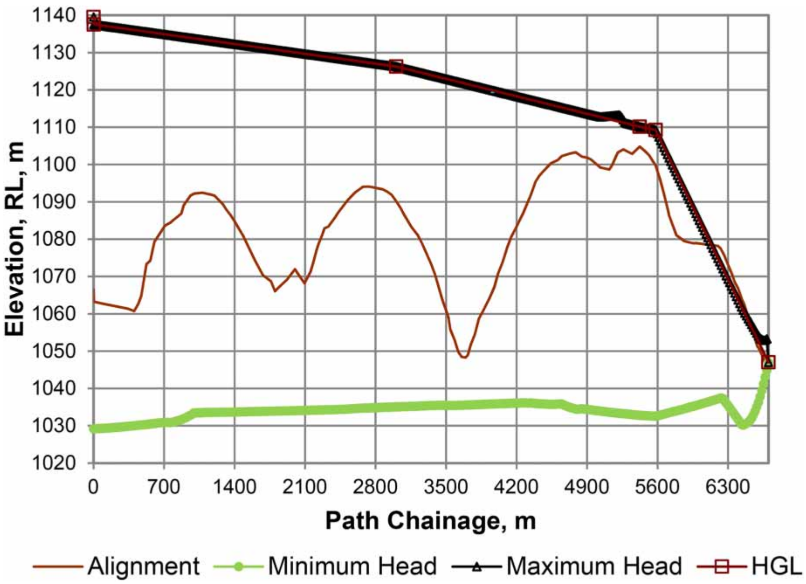


Figure 5 | HGL in the system under power failure of the pumps- entire network.

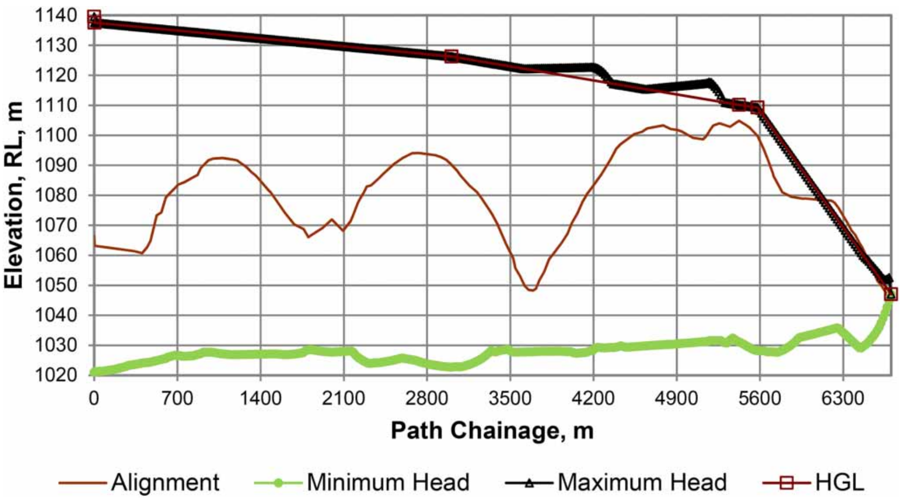


Figure 6 | HGL in the system under power failure of the pumps- 250 mm network isolated.

3.2.2. Sub scenario 2b: when the 250 mm line between node N_2 and N_3 is isolated from main network

Unlike Sub scenario 2a, this scenario experienced considerable upsurge and downsurge, as seen from Figure 6. A considerable upsurge due to column separation is observed around chainages 4,226 m and 5,211 m. It may be also be noted that the transient event occurring due to power failure/sudden pump shut down causes negative pressure in the pipeline well above −10 kPa (negative) or the permissible limits across the majority of the pipe length.

Although pronounced upsurge is observed in this case, the maximum pressure even after the upsurge is well within the working pressure range of the provided pipe. Thus no remedial measures are required to contain the upsurge. However, negative pressure development above the permissible levels may cause cavitations and pipe collapse; therefore, the network must incorporate remedial measures to check this occurrence.

3.3. Scenario 3: surge analysis on baseline network with 7 air valves

Further, the surge model is simulated after incorporation of initially proposed seven (7) double acting air valves of 80 mm diameter across the network. This scenario is further subdivided into two scenarios as Scenario 3a and Scenario 3b.

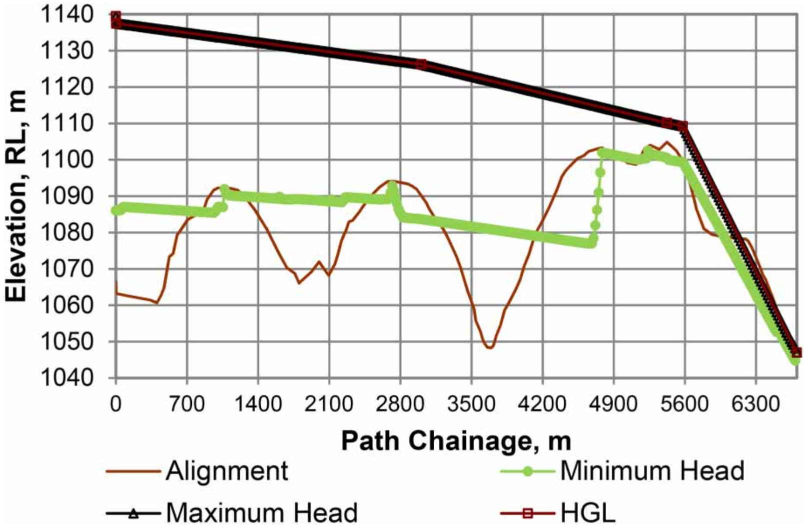


Figure 7 | HGL for power failure of both pumps in the system.

3.3.1. Scenario 3a: transient analysis network with 7 Air valves at the proposed location with 250 mm line isolated

This scenario simulates the baseline network with originally proposed seven (7 no.) double acting vacuum breaker air valves as described in Table 1. Figure 7 shows HGL for power failure of both pumps in the system with 7 air valves at the proposed location with 250 mm line isolated.

As evident from Figure 7, the provision of the proposed 7 no. valves in the network were able to contain the downsurge across the majority of the network but significant downsurge is still present in between chainages 4,200 m and 4,900 m. Thus the originally proposed 7 air valve configuration is not sufficient to contain the surge effect in the pipeline and hence further protection is to be provided.

3.3.2. Scenario 3b: transient analysis network with 7 air valves at the proposed location with 250 mm line included

This model analyses the surge when in addition to the above a parallel line of 250 mm line between node N_2 and N_3 is active (Figure 8). The location of air valves remains the same as in Scenario 2b.

Similar to Scenario 3a, downsurge is present in stretch 4,200–4,900, although the magnitude is lesser but is still over −10 kPa (permissible limits), hence this case further corroborates the need to reposition the proposed air valves.

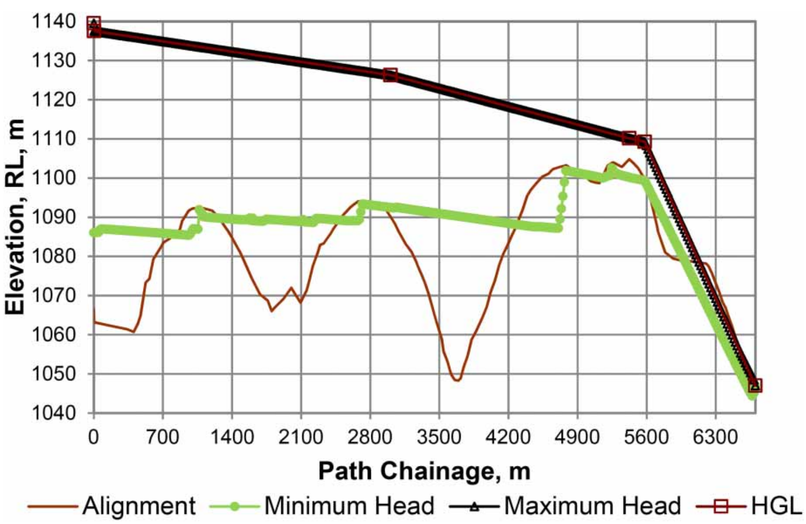


Figure 8 | HGL for power failure of both pumps in the system.

3.4. Scenario 4: with modified air valve location

As inferred in the results of transient analysis for the baseline scenario, Scenario 1 and Scenario 2, power failure of the pump causes negative pressure in the system well below the permissible limits, which can cause cavitation and pipe collapse. Therefore protection is required as most of the pipeline has pressure of the order of vapor pressure below −10 kPa.

This scenario is simulated with slightly different positioning of air cushion valves of 80 mm orifice diameter from Scenario 2. Based on the location of summits and negative pressure formation, five air valves are proposed to contain the effect of downsurge on the entire network.

The altered location of air valves from scenario 2 is provided below with chainages in Table 3. Detailed description and installation guide for the air valves are present in American Water Works Association (1989) and American Water Works Association (2001) respectively.

This scenario is further subdivided into two sub scenarios; that is, Scenario 4a: when the entire network is operational and Scenario 4b: when the 250 mm line is isolated from the main network.

3.4.1. Sub scenario 4a: when the entire network is operational

The resulting variations of pressure and HGL in the system are shown in Figure 9 above. As depicted in the pressure and HGL graph, due to the revised provisions upsurge in the network is completely contained, and the HGL and pressure line for upsurge and steady state condition perfectly overlap each other. Also, the magnitude of negative pressure encountered earlier is reduced and is now well below −10 kPa.

Thus the suggested preventive measures in this scenario completely counter balance the surge effect and make the entire pipe network function within the permissible pressure range even during the transient event.

3.4.2. Sub scenario 4b: when 250 mm line is isolated from the main network

Figure 10 shows the HGL in the system under power failure of the pumps with changed air valve location –250 mm line isolated. It further corroborates the efficacy of revised positioning of air valves in curbing the effect of down surge on the pipe

Table 3 | Proposed chainages of air cushion valves

S.No	Valve No.	Chainage, m
1	AV1	1,085
2	AV2	2,680
3	AV3	4,430
4	AV4	4,787
5	AV5	5,432

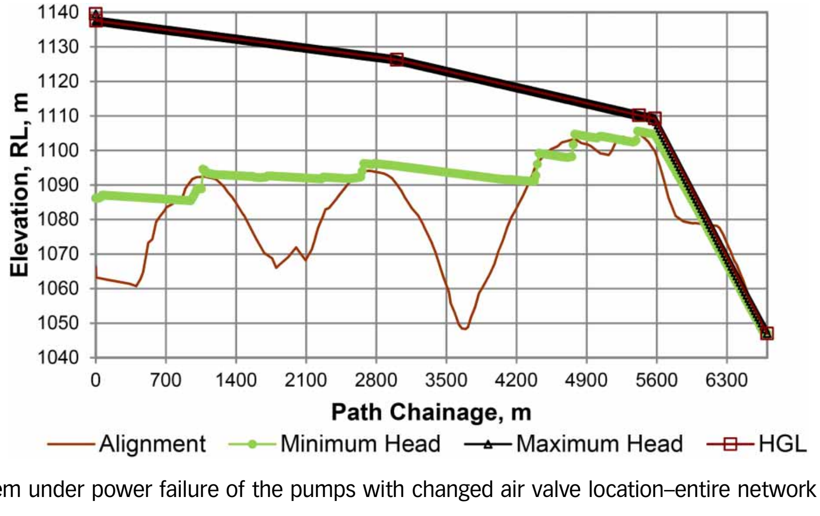


Figure 9 | HGL in the system under power failure of the pumps with changed air valve location–entire network functional.

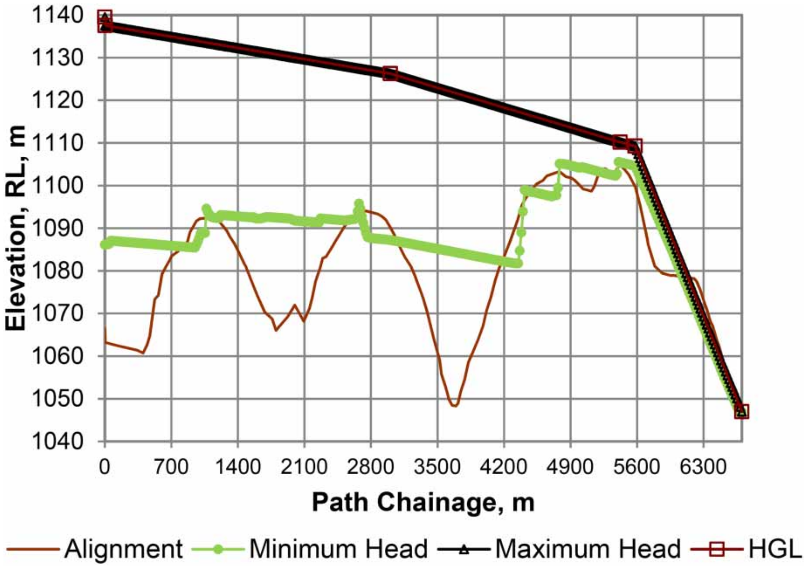


Figure 10 | HGL in the system under power failure of the pumps with changed air valve location –250 mm line isolated.

network. The revised positioning of the air valve is equally effective in containing the downsurge pressure in the pipe network well below -10 kPa.

3.5. Scenario 5: single pump fails

To double check the suggested provisions, a scenario has been further analyzed where only one pump fails due to some unforeseen reason, and again the results obtained are well within the permissible operating range. This scenario is further divided into two sub scenarios; that is, when the entire network is operational and when the 250 mm line is isolated.

3.5.1. Sub scenario 5a: single pump failure- entire network operational

This scenario again corroborates the finding of parent Scenario 4. It can be seen from Figure 11 that pressure and HGL lines are above the entire line that operates within the steady state conditions and permissible negative pressure range as desired.

3.5.2. Sub scenario 5b: single pump failure- 250 mm line isolated

The findings of sub scenario 5b (Figure 12) are similar to sub scenario 5a. This proves the efficacy of revised air cushion valves positioning in curbing the surge effect on the network under various scenarios.

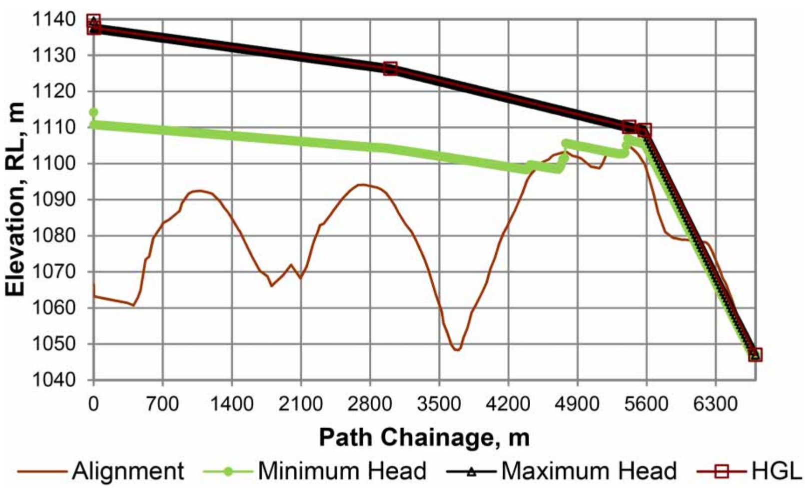


Figure 11 | HGL in the system pump failure- entire network under single operation.

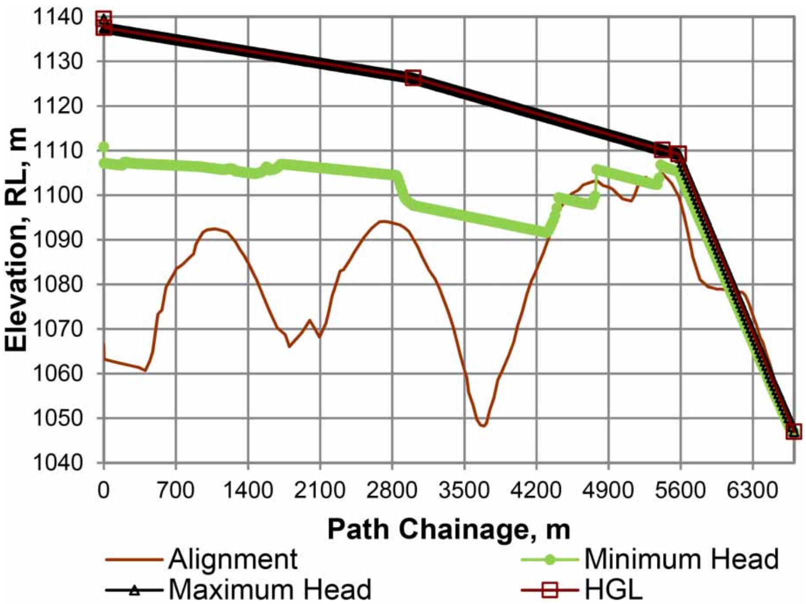


Figure 12 | HGL in the system under single pump failure- 250 mm line isolated.

4. CONCLUSIONS

The water hammering in the pipe due to gradual closure and sudden closure is studied and can be easily demonstrated by the simulated graphs. A total of five scenarios were simulated using different combinations. The simulation results show that the transient pressures in the pipeline exceeded the bar rating of the pipe where the bursts or cavitation may occur for the simulated scenario. Based on the findings, a minimum of five air valves shall be provided to contain the effect of downsurge in the network. However, any additional air valves provided in the network will further improve the network performance. This study will serve as the basis for the field engineers designing the pipeline network as a water conveyance system.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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